Main Points

1) Making demographic projection more realistic
-- improvement #1: adding age structure
-- population projection with matrices

Prereading: Tues 12 Sep = NA; Thurs 14 Sep = Gilbert et al 2016

Quiz #1 will be available today at 5pm in WyoCourses. It covers 29 Aug – 7 Sep.

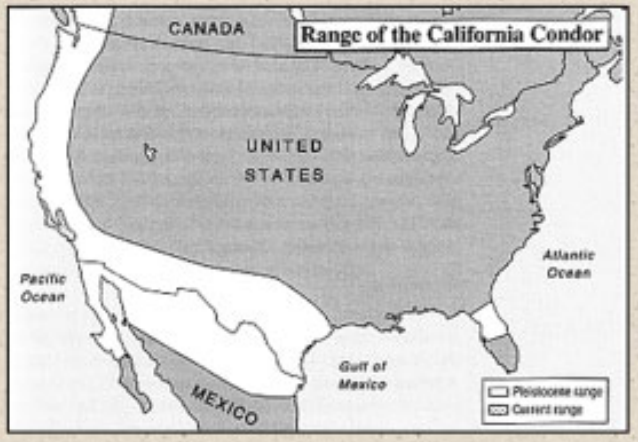
Terms: age/stage structured matrix (Leslie/Leftkovitch matrix), stable age/stage distribution

Punchline #1: California condors are a Pleistocene relict. To illustrate demographic stochasticity, we went through an example of calculating survival probabilities of a condor pair.

PVA of California Condor

TABLE 2.1 Possible outcomes of releasing a pair of condors, each with a survival probability of 85%

Event	*Fate of female condor*	*Fate of male condor*	*Probability*
Both survive	Live ($p = 0.85$)	Live ($p = 0.85$)	$0.85 \times 0.85 = 0.7225$
One bird survives	Live ($p = 0.85$)	Die ($p = 0.15$)	$0.85 \times 0.15 = 0.1275$
	Die ($p = 0.15$)	Live ($p = 0.85$)	$0.15 \times 0.85 = 0.1275$
Neither survives	Die ($p = 0.15$)	Die ($p = 0.15$)	$0.15 \times 0.15 = 0.0225$



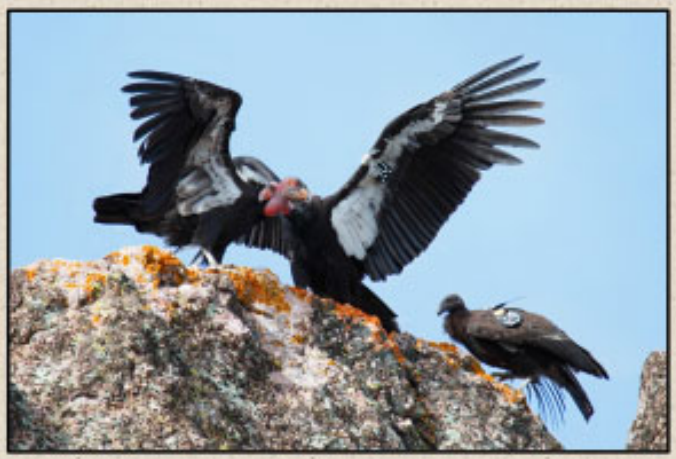
Morris and Doak. 2004. 14

Punchline #2: geometric means of λ are more useful than averages in projecting population size through time, because we are dealing with probabilities. We went through an exercise to project condor population size.

Discussion: imagine a population of reintroduced condors that you've monitored since 2020, with these data:

$$\lambda_{2020} = 1.0, \lambda_{2021} = 1.05, \lambda_{2022} = 1.05, \lambda_{2023} = 0.82$$

Calculate the geometric mean, and use it to project a starting population of 30 ($N_0 = 30$) 20 years ($t = 20$) into the future.



$$N_t = N_0\lambda^t$$

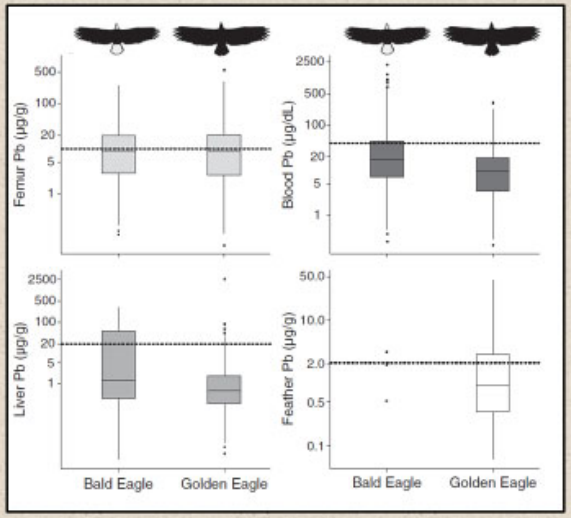
Geometric mean =

$$\lambda_G = \prod_{i=1}^{n} x_i^{p_i}$$

28

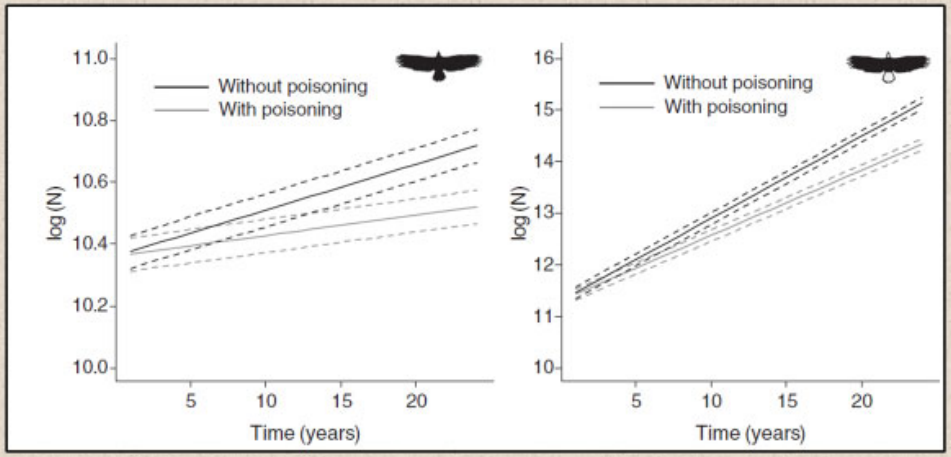
Punchline #3: Slabe et al used age structured demographic projection (AKA demographic PVA) to show that bald and golden eagle populations are suppressed through lead poisoning.

Projecting populations, improvement #1: demographic PVA



Slabe et al. 2022. 31

Projecting populations, improvement #1: demographic PVA

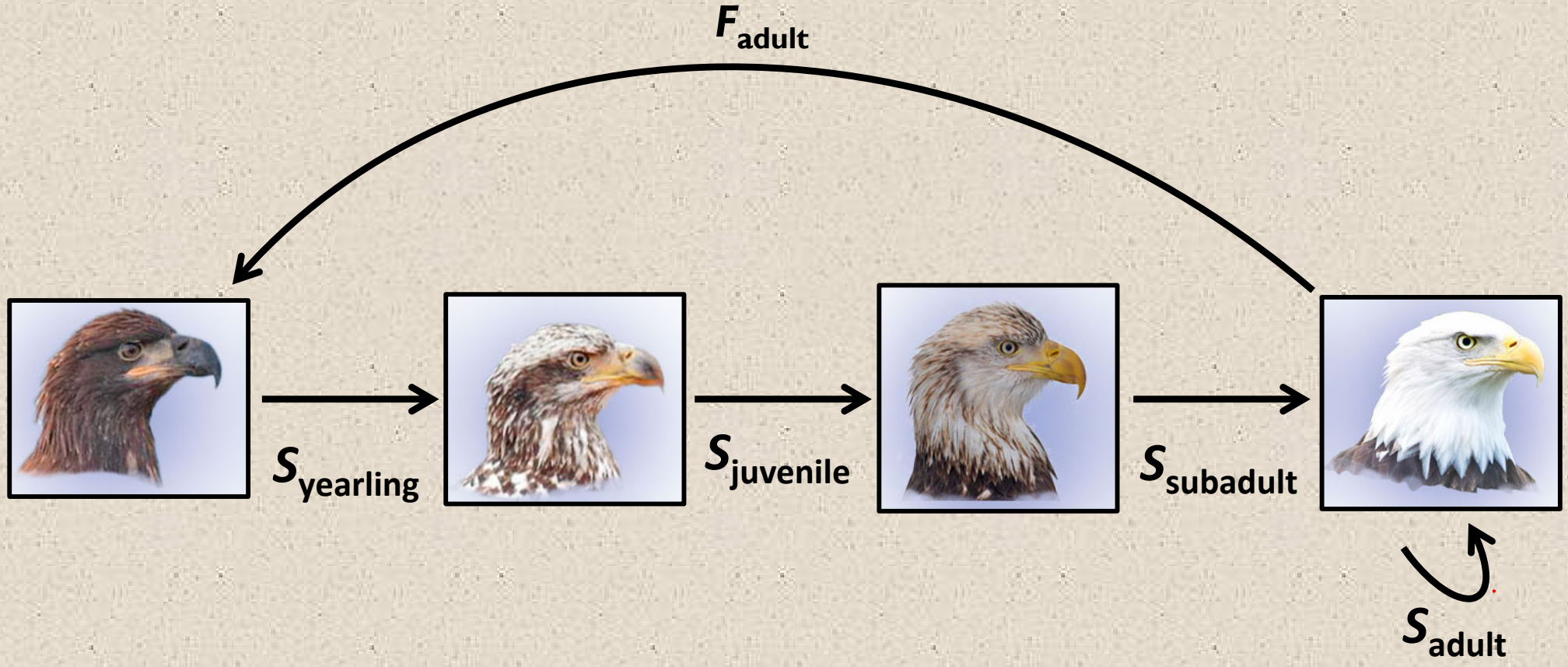


Slabe et al. 2022. 36

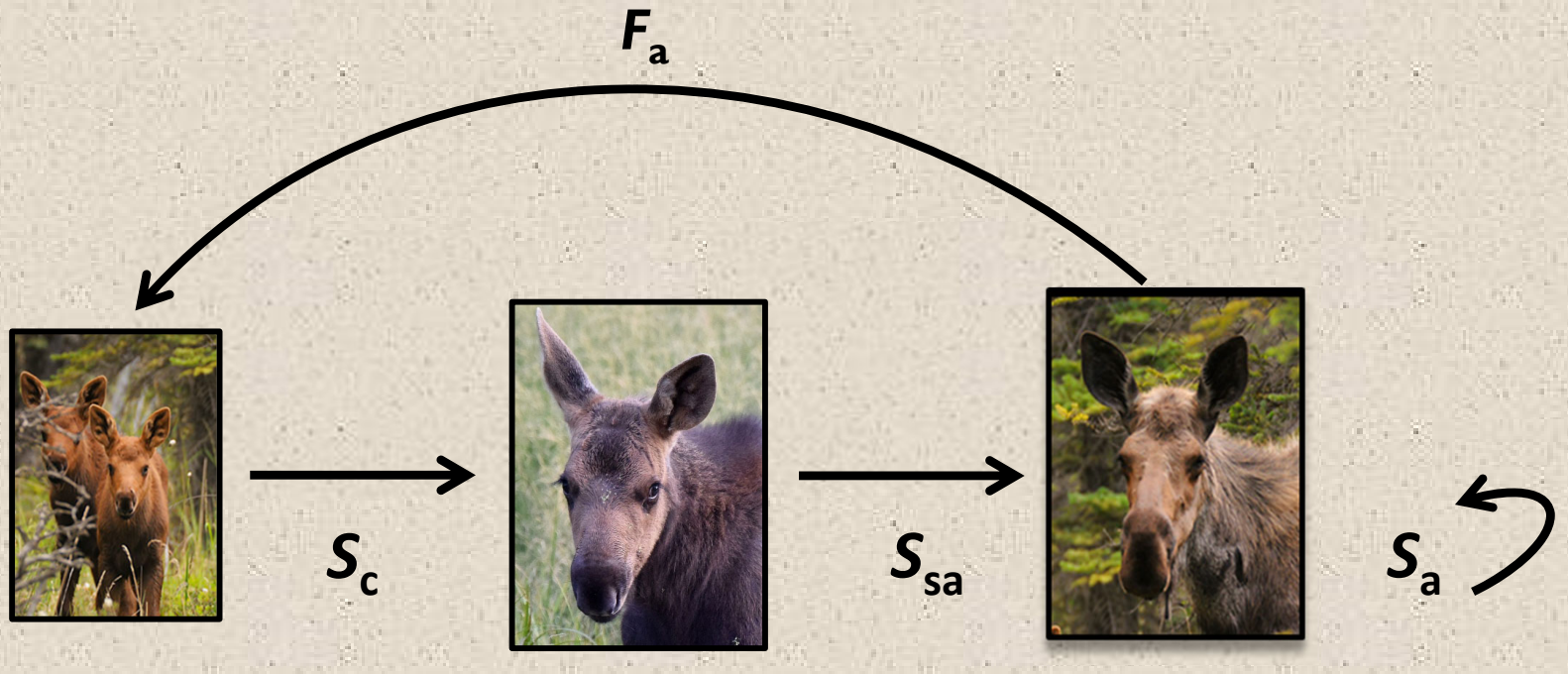
Projecting populations, improvement #1: demographic PVA

- **<u>age/stage structured matrix (Leslie/Leftkovitch matrix)</u> = matrix of age-dependent vital rates used to project populations.**

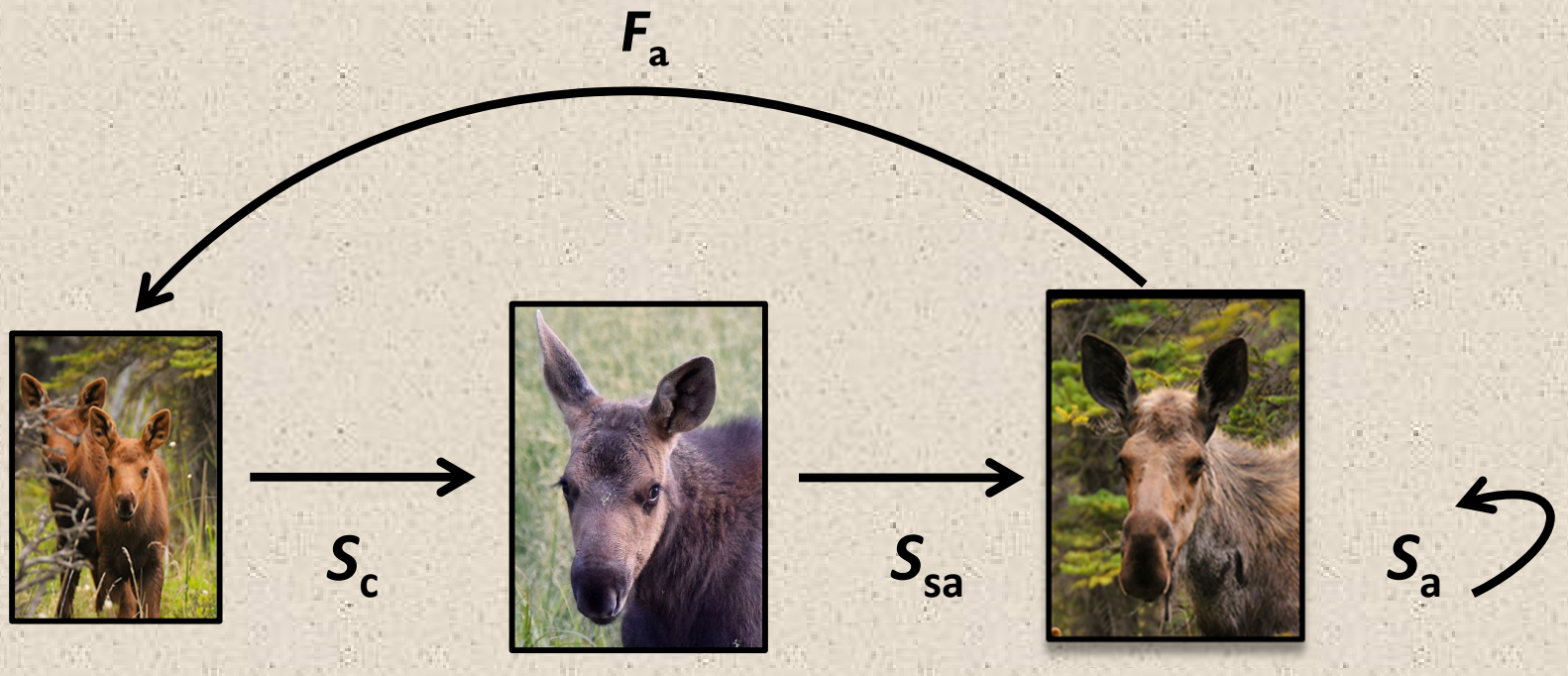
Projecting populations, improvement #1: demographic PVA



Matrix modeling to conduct demographic PVA

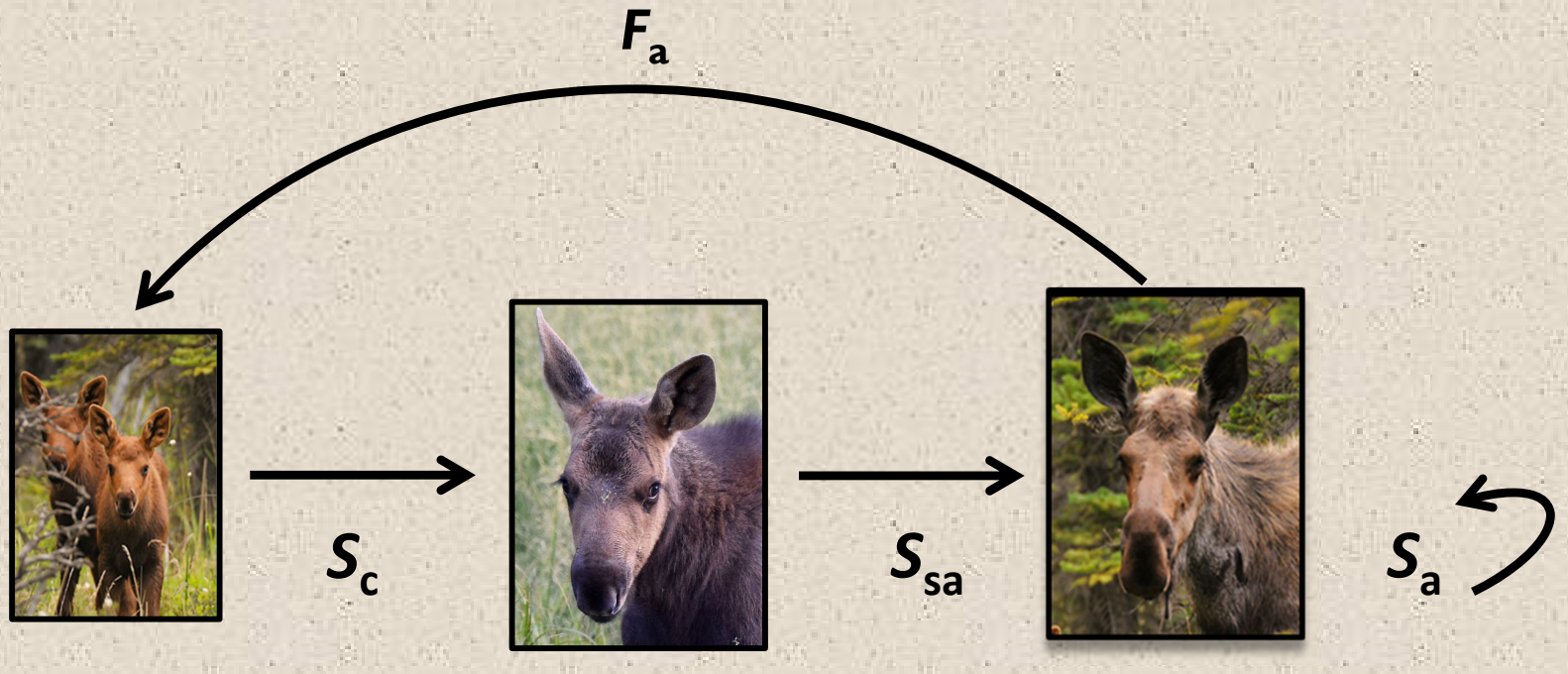


Matrix modeling to conduct demographic PVA



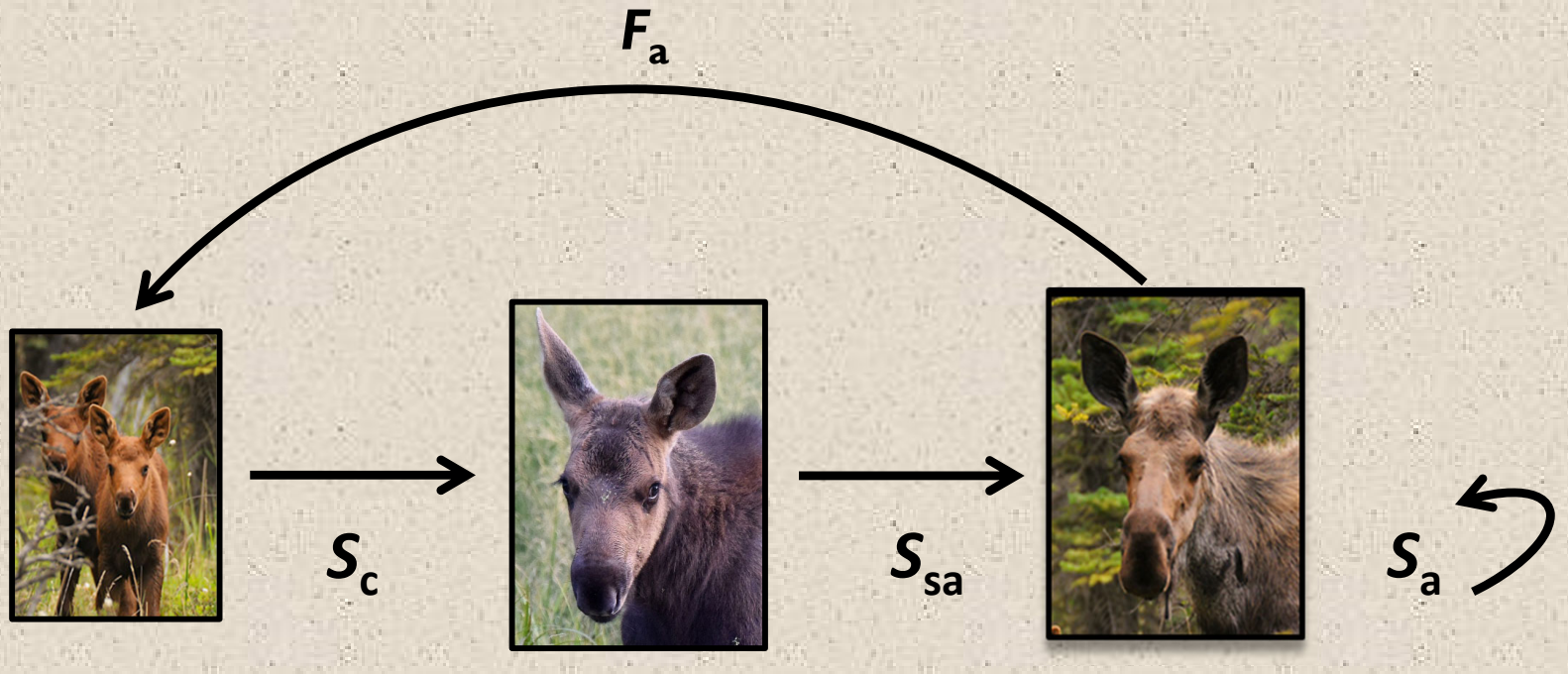
$$\begin{bmatrix} 0 & 0 & S_a F_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



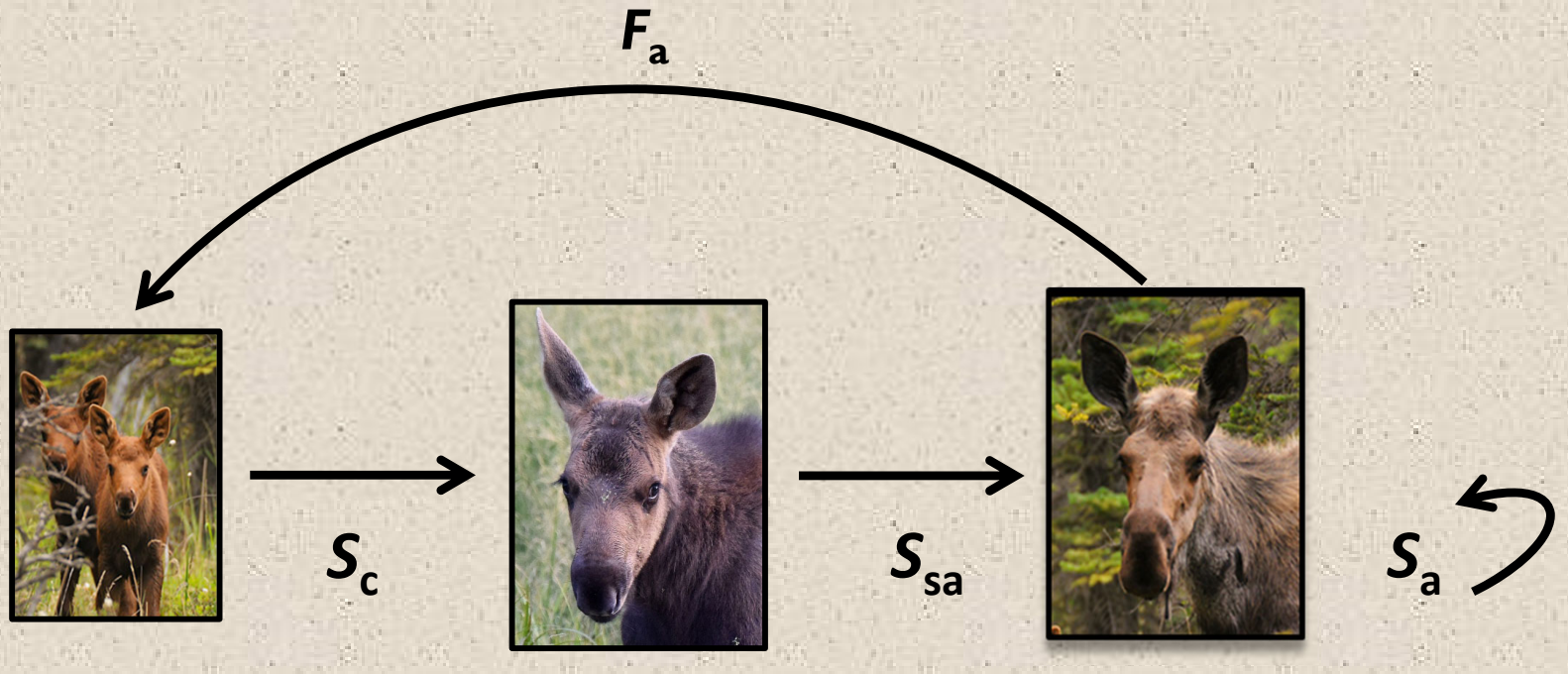
$$\begin{bmatrix} 0 & 0 & S_a F_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



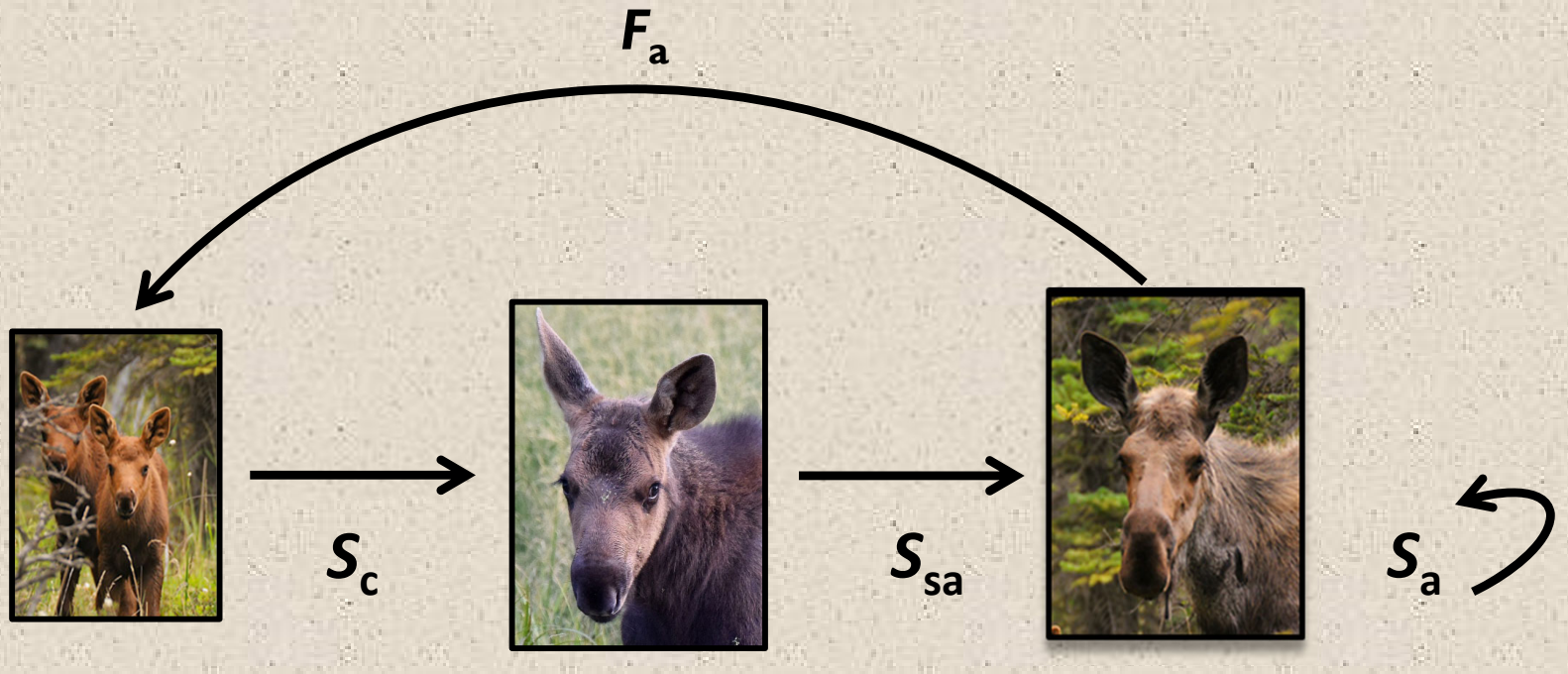
$$\begin{bmatrix} 0 & 0 & S_aF_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix} = \begin{bmatrix} 0*n_c + 0*n_{sa} + S_aF_a*n_a \\ \\ \\ \end{bmatrix} = \begin{bmatrix} n_{c1} \\ \\ \\ \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



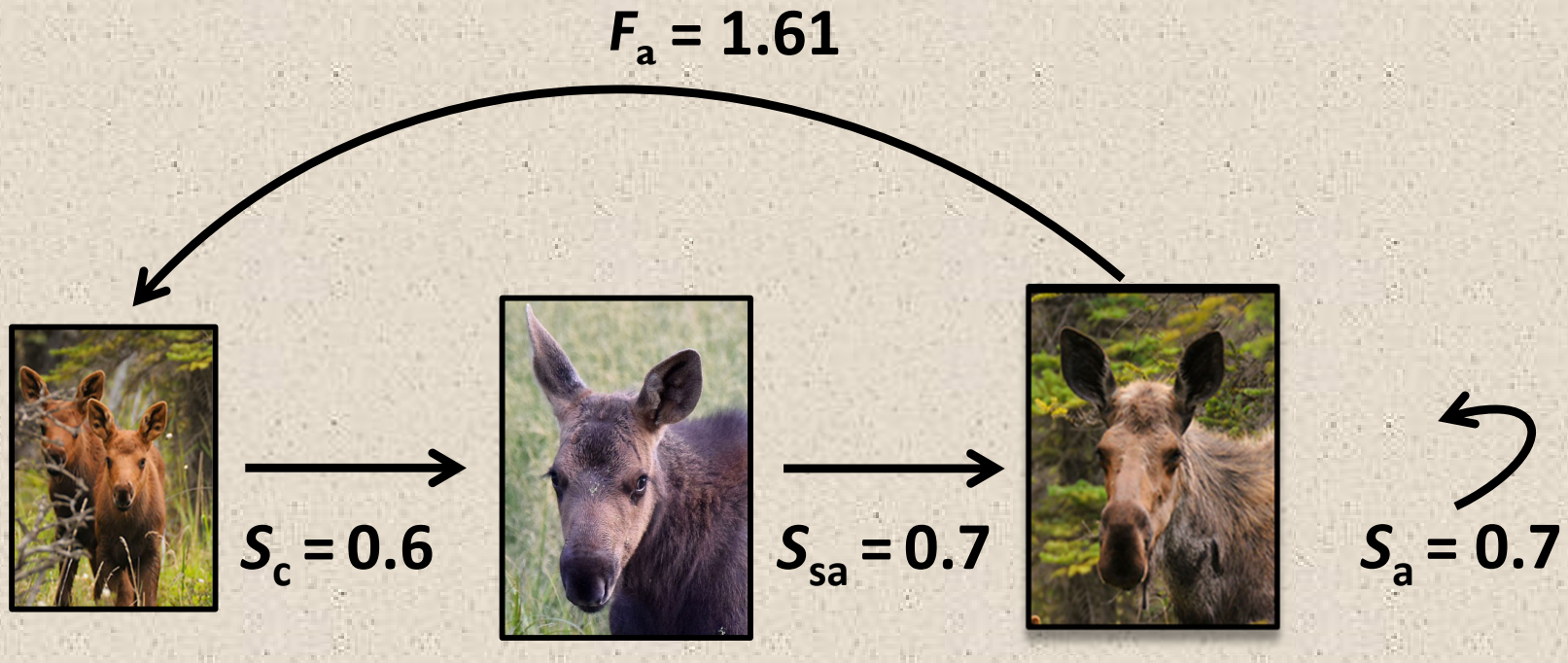
$$\begin{bmatrix} 0 & 0 & S_a F_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix} = \begin{bmatrix} 0*n_c + 0*n_{sa} + S_a F_a * n_a \\ S_c * n_c + 0*n_{sa} + 0*n_a \end{bmatrix} = \begin{bmatrix} n_{c1} \\ n_{sa1} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



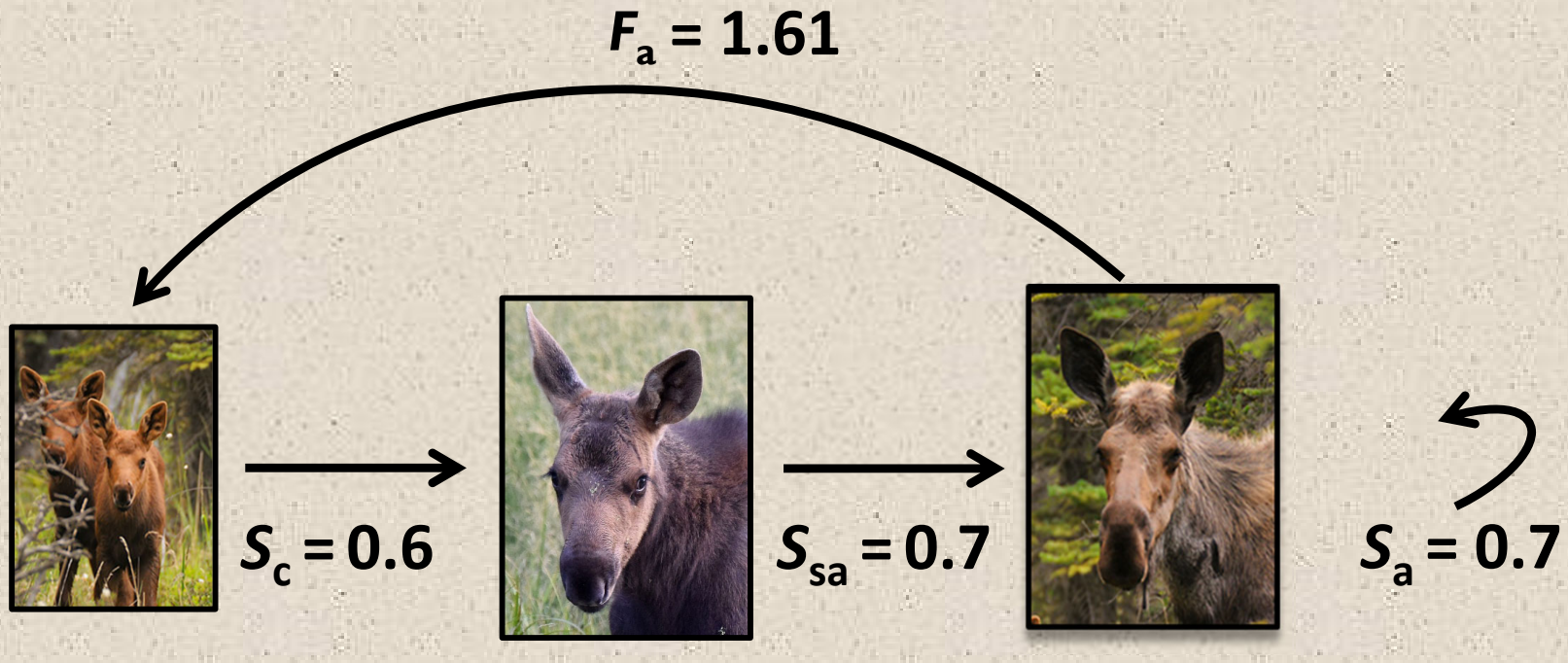
$$\begin{bmatrix} 0 & 0 & S_a F_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix} = \begin{bmatrix} 0*n_c + 0*n_{sa} + S_a F_a * n_a \\ S_c * n_c + 0*n_{sa} + 0*n_a \\ 0*n_c + S_{sa} * n_{sa} + S_a * n_a \end{bmatrix} = \begin{bmatrix} n_{c1} \\ n_{sa1} \\ n_{a1} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



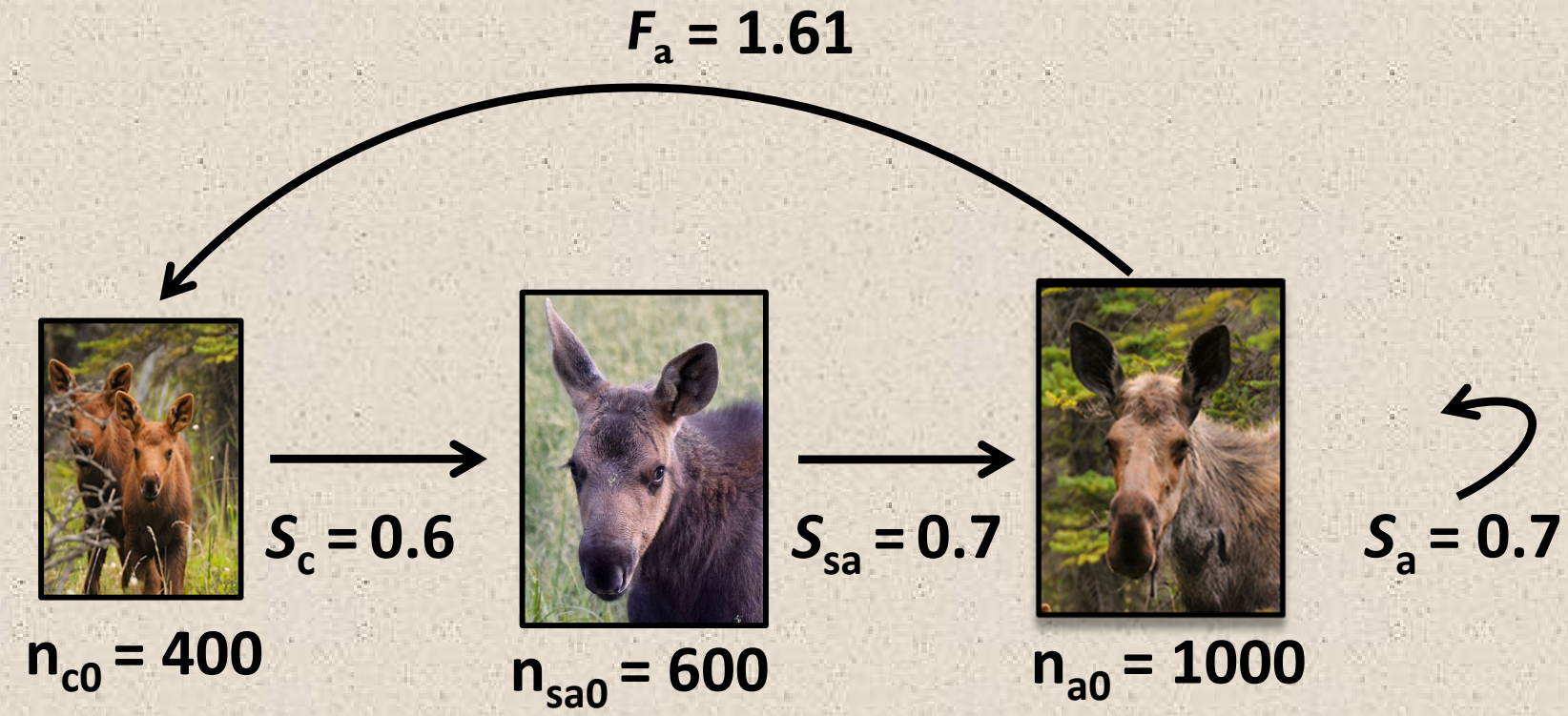
$$\begin{bmatrix} 0 & 0 & S_aF_a \\ S_c & 0 & 0 \\ 0 & S_{sa} & S_a \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix} = \begin{bmatrix} 0*n_c + 0*n_{sa} + S_aF_a*n_a \\ S_c*n_c + 0*n_{sa} + 0*n_a \\ 0*n_c + S_{sa}*n_{sa} + S_a*n_a \end{bmatrix} = \begin{bmatrix} n_{c1} \\ n_{sa1} \\ n_{a1} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix}$$

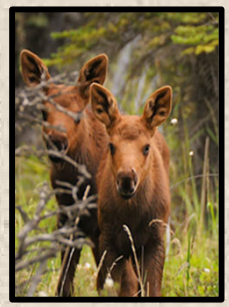
Matrix modeling to conduct demographic PVA



$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} n_{c0} \\ n_{sa0} \\ n_{a0} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA

$F_a = 1.61$



$S_c = 0.6$

$S_{sa} = 0.7$

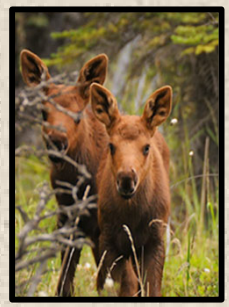
$S_a = 0.7$

$n_{c0} = 400$ $n_{sa0} = 600$ $n_{a0} = 1000$

$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*n_c + 0*n_{sa} + S_a F_a * n_a \\ S_c * n_c + 0*n_{sa} + 0*n_a \\ 0*n_c + S_{sa} * n_{sa} + S_a * n_a \end{bmatrix}$$

Matrix modeling to conduct demographic PVA

$F_a = 1.61$



$S_c = 0.6$

$S_{sa} = 0.7$

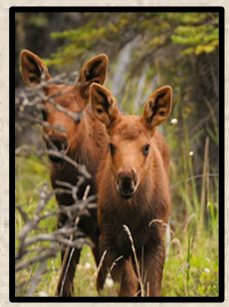
$S_a = 0.7$

$n_{c0} = 400$ $n_{sa0} = 600$ $n_{a0} = 1000$

$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*400 + 0*600 + 1.13*1000 \\ S_c*n_c + 0*n_{sa} + 0*n_a \\ 0*n_c + S_{sa}*n_{sa} + S_a*n_a \end{bmatrix}$$

Matrix modeling to conduct demographic PVA

$F_a = 1.61$



$S_c = 0.6$

$S_{sa} = 0.7$

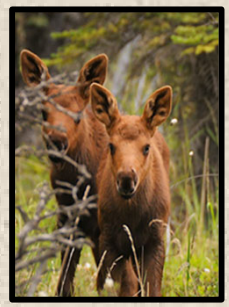
$S_a = 0.7$

$n_{c0} = 400$ $n_{sa0} = 600$ $n_{a0} = 1000$

$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*400 + 0*600 + 1.13*1000 \\ 0.6*400 + 0*600 + 0*1000 \\ 0*n_c + S_{sa}*n_{sa} + S_a*n_a \end{bmatrix}$$

Matrix modeling to conduct demographic PVA

$F_a = 1.61$



$S_c = 0.6$

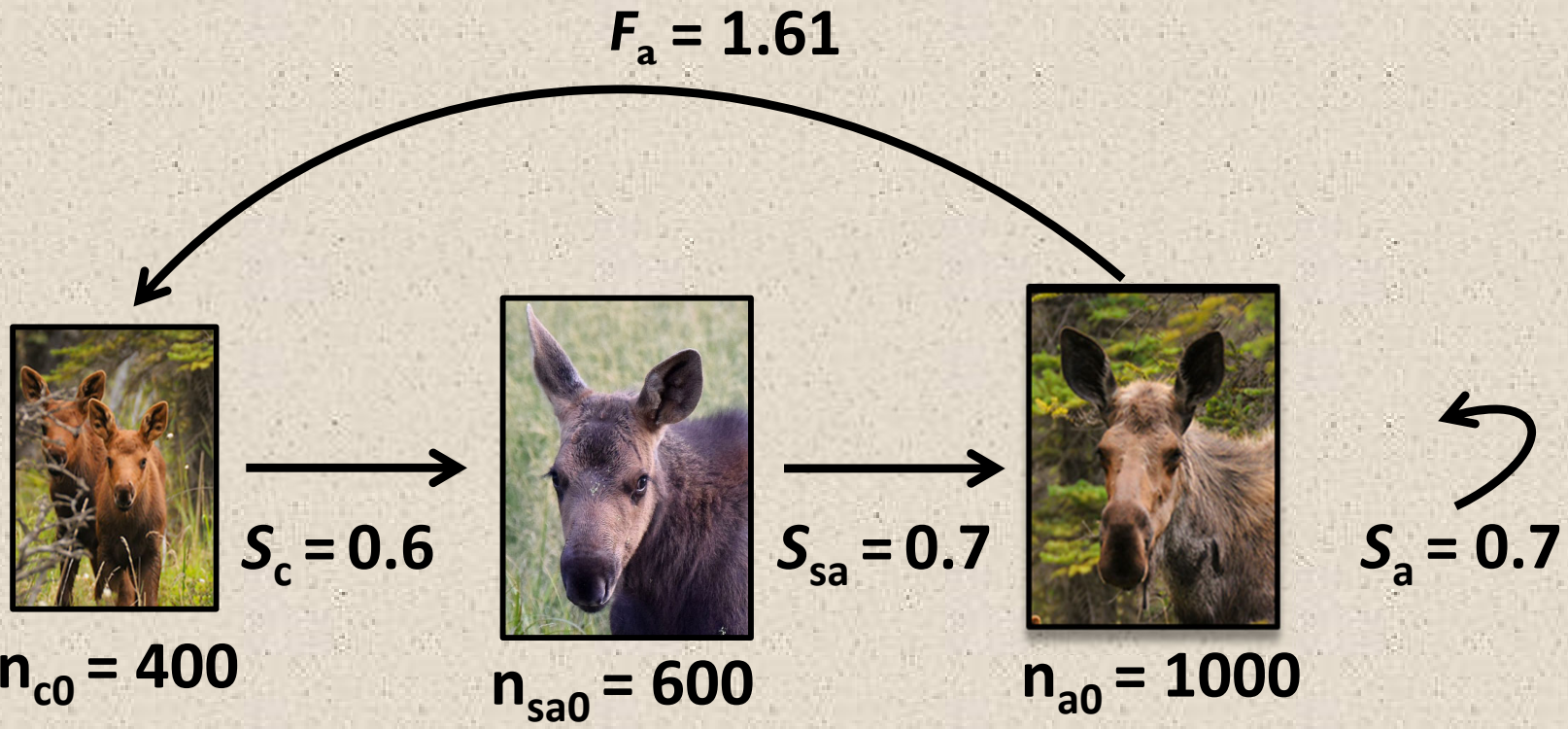
$S_{sa} = 0.7$

$S_a = 0.7$

$n_{c0} = 400$ $n_{sa0} = 600$ $n_{a0} = 1000$

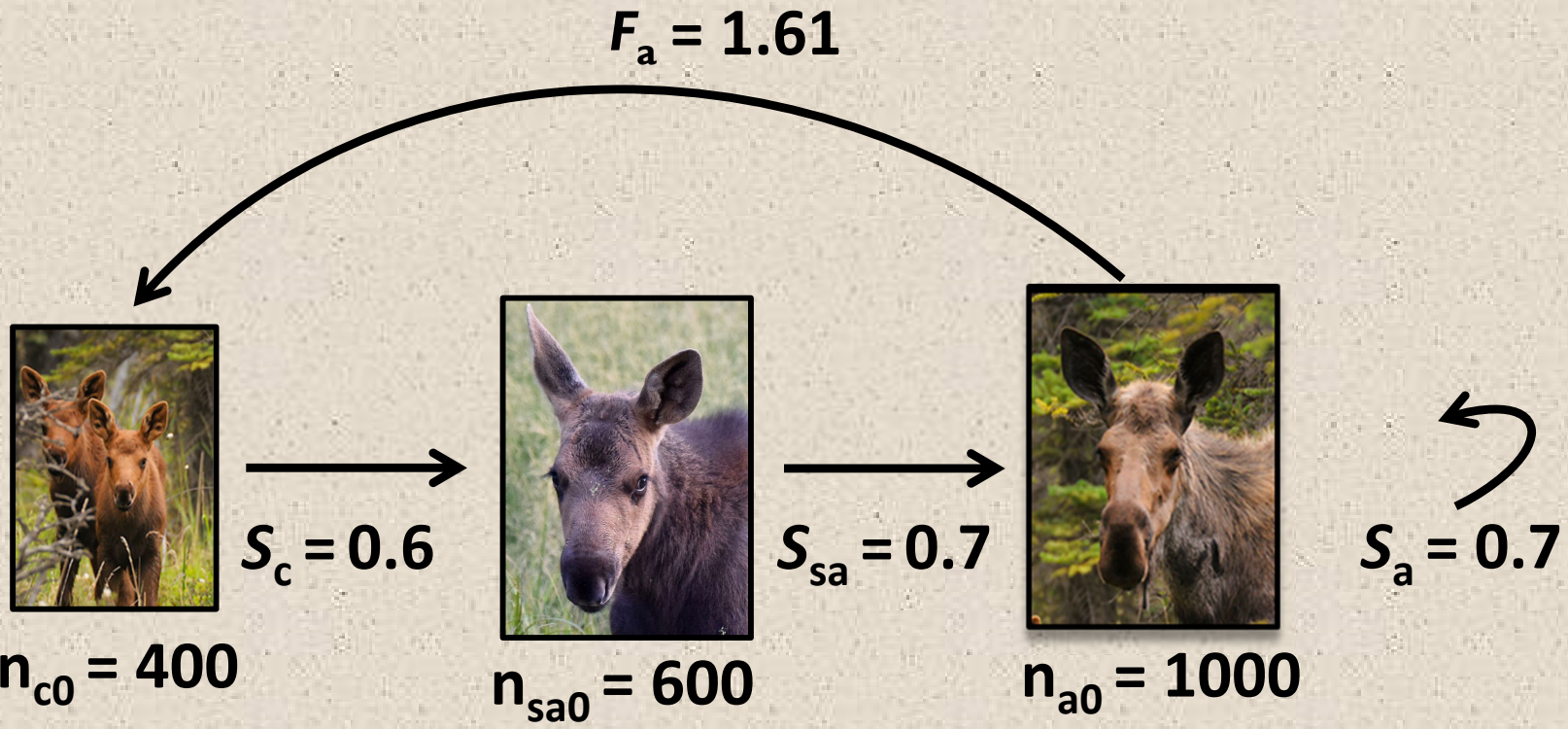
$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*400 + 0*600 + 1.13*1000 \\ 0.6*400 + 0*600 + 0*1000 \\ 0*400 + 0.7*600 + 0.7*1000 \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



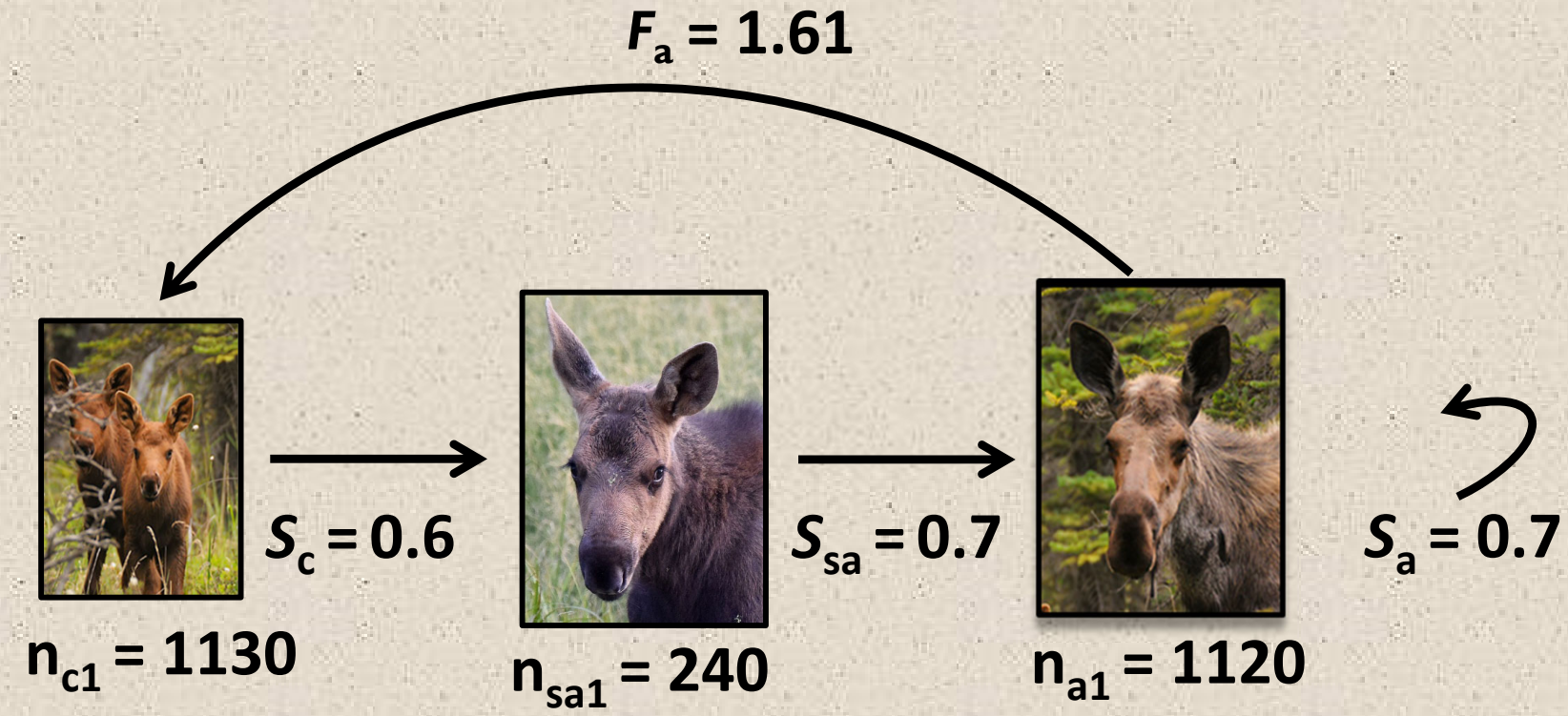
$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*400 + 0*600 + 1.13*1000 \\ 0.6*400 + 0*600 + 0*1000 \\ 0*400 + 0.7*600 + 0.7*1000 \end{bmatrix} = \begin{bmatrix} n_{c1} \\ n_{sa1} \\ n_{a1} \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = \begin{bmatrix} 0*400 + 0*600 + 1.13*1000 \\ 0.6*400 + 0*600 + 0*1000 \\ 0*400 + 0.7*600 + 0.7*1000 \end{bmatrix} = \begin{bmatrix} 1130 \\ 240 \\ 1120 \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



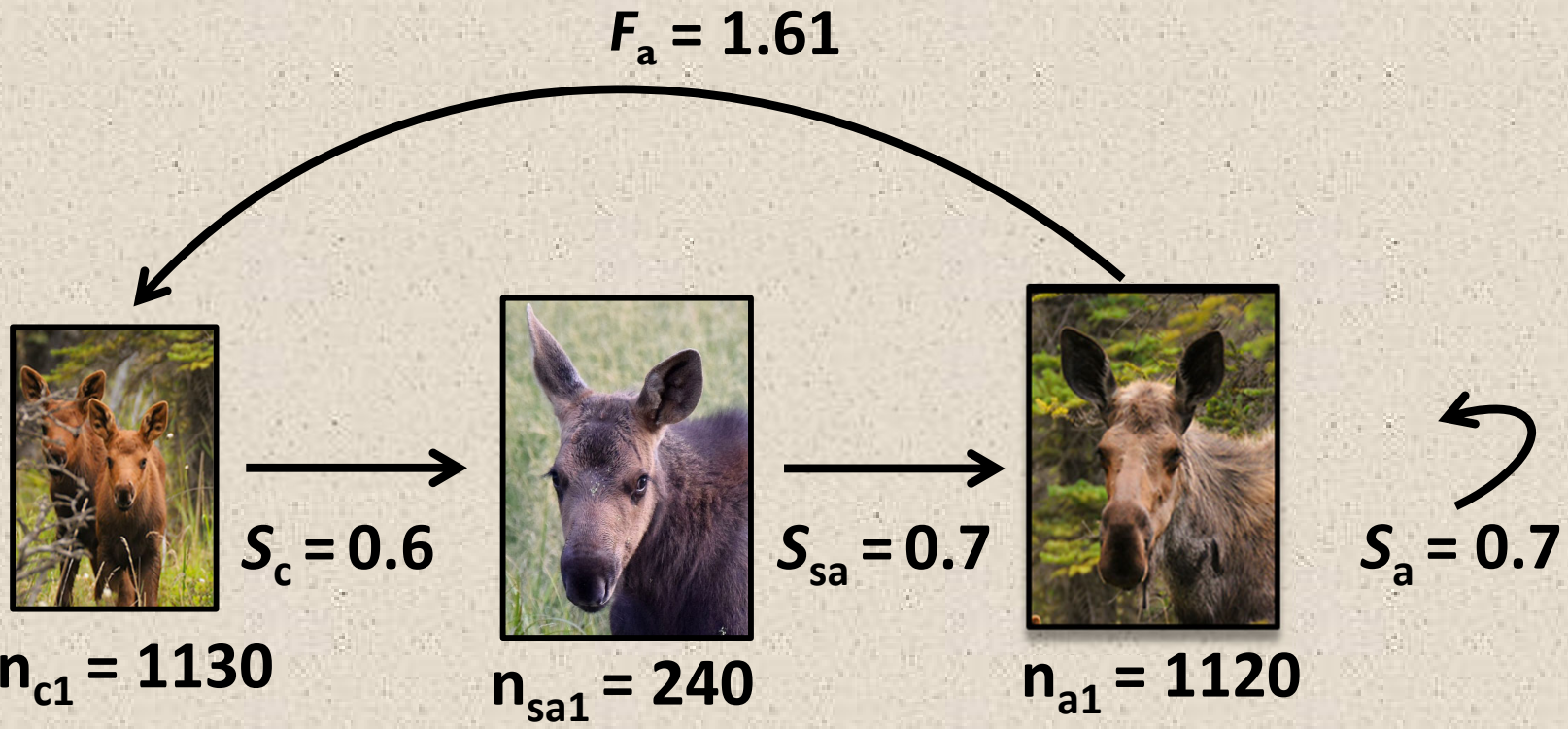
Year 0

$$\begin{bmatrix} 400 \\ 600 \\ 1000 \end{bmatrix} = 2000$$

Year 1

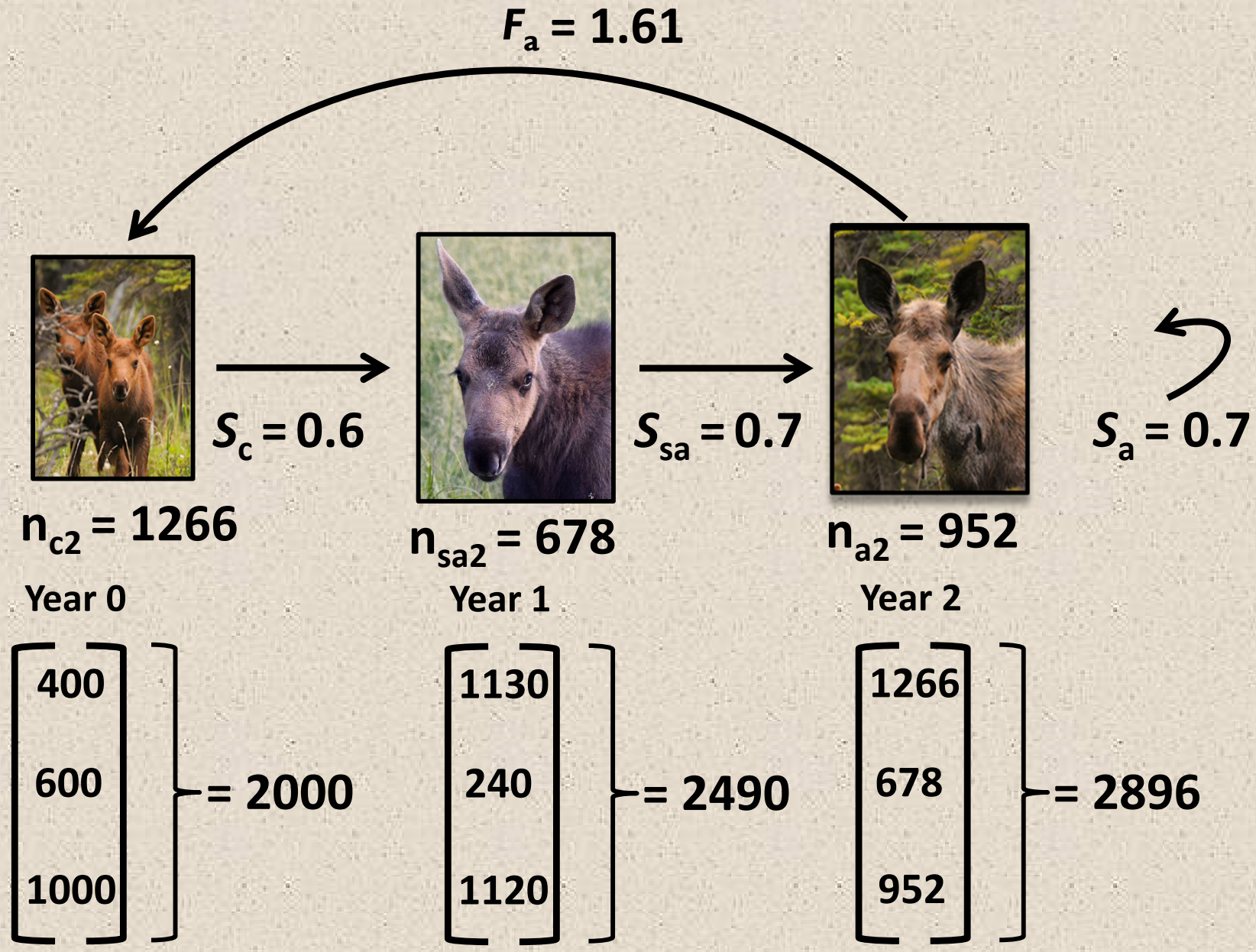
$$\begin{bmatrix} 1130 \\ 240 \\ 1120 \end{bmatrix} = 2490$$

Matrix modeling to conduct demographic PVA



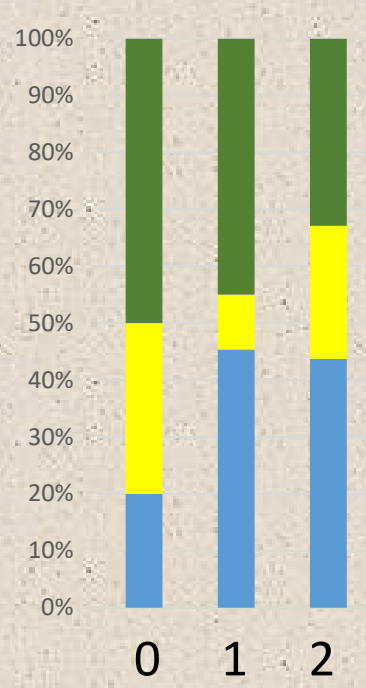
$$\begin{bmatrix} 0 & 0 & 1.13 \\ 0.6 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{bmatrix} * \begin{bmatrix} 1130 \\ 240 \\ 1120 \end{bmatrix} = \begin{bmatrix} 0+0+1266 \\ 678+0+0 \\ 0+168+784 \end{bmatrix} = \begin{bmatrix} 1266 \\ 678 \\ 952 \end{bmatrix}$$

Matrix modeling to conduct demographic PVA



Matrix modeling to conduct demographic PVA

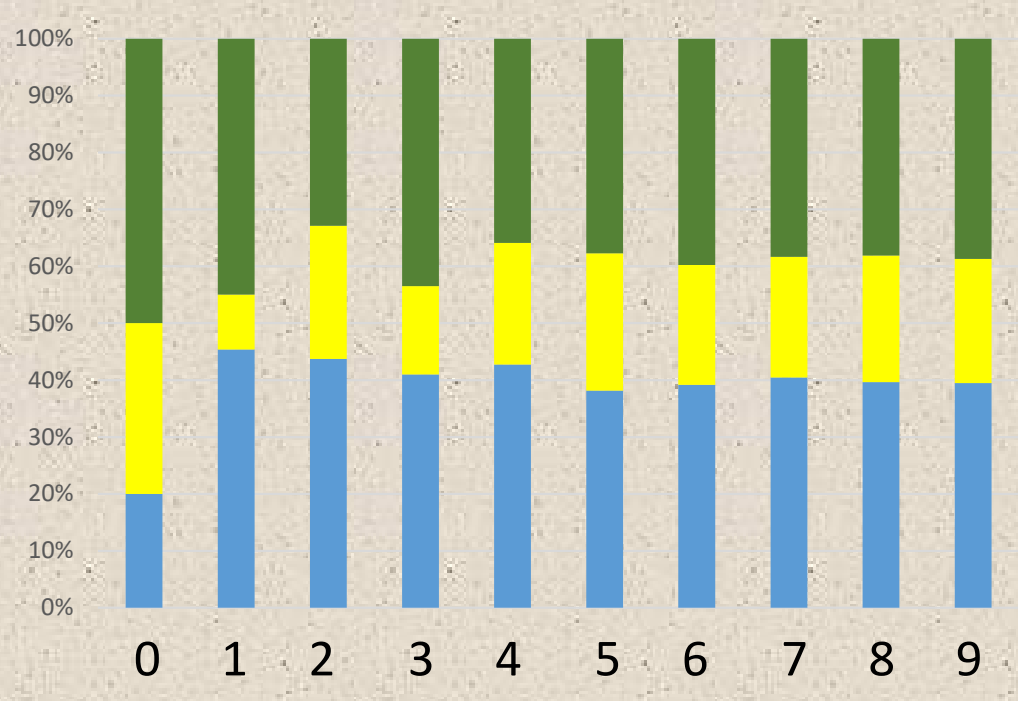
Year 0	Year 1	Year 2
400	1130	1266
600	240	678
1000	1120	952



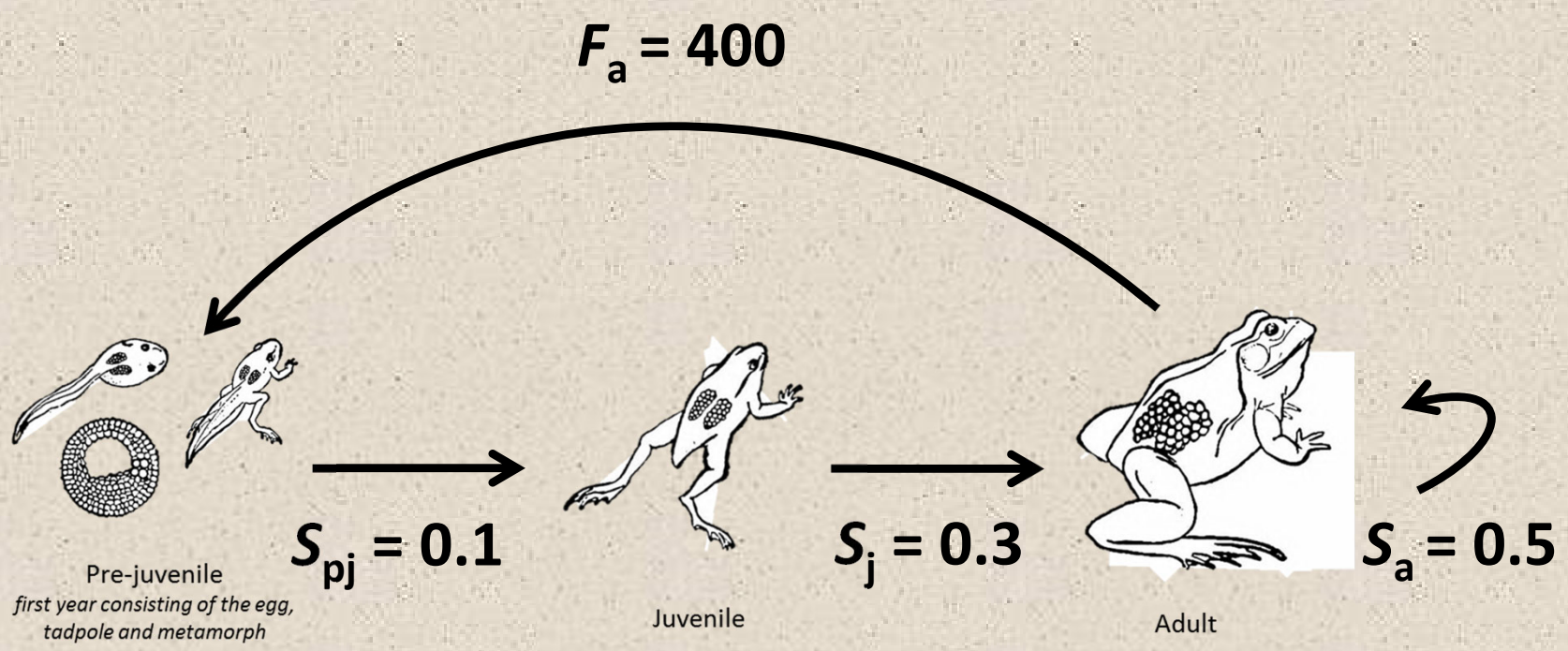
Matrix modeling to conduct demographic PVA

- **<u>stable age (or stage) distribution</u> = the age (or stage) structure toward which the population tends over time, assuming vital rates do not change.**

Year 0	Year 1	Year 2
400	1130	1266
600	240	678
1000	1120	952



<u>Discussion:</u> below is the life cycle and numbers of the Wyoming toad in 2020 in Mortenson National Wildlife Refuge. Project a starting population size of 500 pre-juveniles, 50 juveniles, and 30 adults 1 year into the future, and calculate λ.



<u>Discussion:</u> below is the life cycle and numbers of the Wyoming toad in 2020 in Mortenson National Wildlife Refuge. Project a starting population size of 500 pre-juveniles, 50 juveniles, and 30 adults 1 year into the future, and calculate λ.

Remember:

-- inner dimensions of the demographic matrix and vector should match

-- the number of arrows in the life cycle is the number of non-zero matrix elements

-- the demographic matrix is square, with the same number of rows and columns as age classes in the life cycle